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Model of Scalar Mesons

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Abstract

A model containing a nonet of scalar mesons S, a nonet of pseudoscalar mesons P, and a nonet of baryons is constructed where the mesons enter in the form of the matrix $M = e^{i(S+iP)}$. Several Lagrangians are introduced such that the mesons get their physical masses, and the decay widths of the scalar mesons are calculated. The model satisfies generalized PCAC. It is found that the coupling constants of the mesons $\varepsilon(700)$ and $\varepsilon'(1060)$ to pions and nucleons satisfy the relations:

$$\frac{|G_{\varepsilon NN}|}{|G_{\varepsilon\pi\pi}|} = 1{\cdot}4, \quad \frac{|G_{\varepsilon' NN}|}{|G_{\varepsilon'\pi\pi}|} = 0{\cdot}89, \quad \frac{|G_{\varepsilon\pi\pi}\, G_{\varepsilon NN}|}{4\pi} = 4{\cdot}9, \quad \text{and} \quad \frac{|G_{\varepsilon'\pi\pi}\, G_{\varepsilon' NN}|}{4\pi} = 0{\cdot}34.$$

1. *Introduction*

The algebraic formulation of chiral $SU(3)_L \otimes SU(3)_R$ symmetry known as current algebra (Gell-Mann, 1962), has offered much in the theory of elementary particles. It was found later that the results of current algebra can be obtained also from effective Lagrangians (Weinberg, 1967), and for this reason they were used extensively in the last few years (Gasiorowicz & Geffen, 1969). The fields which are encountered often in phenomenological Lagrangians are a nonet of pseudoscalar fields, a nonet of scalar fields and a nonet of baryon fields. In this paper we construct a model containing these nonets.

In Section 2 we assume that the mesons enter in the form of a matrix M which is a function of a combination of the scalar and pseudoscalar meson fields, and which transforms as the $(3_L, 3_R^*)$ representation of $SU(3)_L \otimes SU(3)_R$. From this matrix the kinetic energy Lagrangian is constructed, and also several mass Lagrangians. Symmetry breaking is introduced through the matrix λ_8. All scalar and pseudoscalar meson fields can get arbitrary masses. The mixing angle of the $I = 0$ members

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of the pseudoscalar nonet is determined in terms of the masses of this nonet, and the same thing happens for the scalar nonet. The model satisfies generalized PCAC (partially conserved axial vector current). The terms of the expansion of our Lagrangians which have three or more fields are taken as interaction Lagrangian terms. The interaction Lagrangian containing a scalar field and two pseudoscalar fields is calculated.

In Section 3 the decay widths $\Gamma(\varepsilon \to \pi\pi)$, $\Gamma(\delta \to \pi\eta)$, $\Gamma(\varepsilon' \to \bar{K}K)$, $\Gamma(K' \to K\pi)$ and $\Gamma(\eta' \to \eta + 2\pi)$ are estimated. Also the S-wave $\pi\pi$, πK and KK scattering lengths are calculated. Most scattering lengths are very close to those given by the usual effective Lagrangian approach (Cronin, 1967), i.e. the contribution of the scalar meson exchange is small. The decays of scalar mesons and the $\pi\pi$ scattering lengths have also been calculated on the basis of a linear realization of chiral $SW(3)$ which is broken spontaneously (Schechter & Ueda, 1971; Suzuki *et al.*, 1971).

In Section 4 a nonet of baryons is introduced into the model. Several Lagrangians are added such that the baryons get their physical masses, and the coupling constants of the mesons $\varepsilon(700)$ and $\varepsilon'(1060)$ are calculated. The product of the coupling constants of ε to nucleons and pions agrees with rough experimental estimates (Engels, 1970; Peterson & Pišut, 1972; Ebel *et al.*, 1971), and the ratio of these constants is close to the value found from the coupling of ε to the trace of the energy-momentum tensor (Genz & Steiner, 1971a). Also the product of the coupling constants of ε' to nucleons and pions and the ratio of these constants are close to the corresponding values found from the coupling of ε' to the trace of the energy-momentum tensor (Genz & Steiner, 1971b).

The meson matrix is a Lorentz scalar of conformal weight -1. This allows the introduction of the conformal symmetry in an easy way.

2. *Meson Lagrangians*

Let S and P be two 3×3 matrices representing a nonet of scalar mesons and a nonet of pseudoscalar mesons respectively, and let us consider the combination

$$\Phi = S + iP \tag{2.1}$$

We assume that the above mesons enter only in the form of the matrix M, where

$$M = e^{f\Phi} \tag{2.2}$$

If we omit the scalar fields the above matrix M reduces to the pseudoscalar meson matrix used in phenomenological chiral symmetry models (Cronin, 1967). M is taken to transform as follows under the chiral group $SU(3)_L \otimes SU(3)_R$

$$M_\beta{}^\alpha \sim (3_L, 3_R^*) \tag{2.3}$$

Then the Lagrangian

$$L_{KE} = -\frac{1}{2f^2}\mathrm{Tr}\,(\partial_\mu M\,\partial_\mu M^+) \tag{2.4}$$

is invariant under the group $SU(3)_L \otimes SU(3)_R$.

The transformations $x_\mu \to x_\mu'$ of the conformal group satisfy the relations (Isham *et al.*, 1970)

$$\frac{\partial x_\mu'}{\partial x_\kappa}\frac{\partial x_\nu'}{\partial x_\lambda} g^{\mu\nu} = \left|\det\left(\frac{\partial x'}{\partial x}\right)\right|^{1/2} g^{\kappa\lambda} \tag{2.5}$$

If the fields $\psi(x)$ belong to a linear representation of the inhomogeneous Lorentz group

$$\psi'(x') = D(\Lambda)\,\psi(x) \tag{2.6}$$

then the conformal transformations are represented by (Isham *et al.*, 1970)

$$\psi'(x') = \left|\det\left(\frac{\partial x'}{\partial x}\right)\right|^{-l_\psi/4} D[\Lambda(x)]\,\psi(x) \tag{2.7}$$

where the matrix $\Lambda(x)$ is given by

$$\Lambda_{\mu\nu}(x) = \left|\det\left(\frac{\partial x'}{\partial x}\right)\right|^{-1/4} \frac{\partial x_\mu'}{\partial x^\nu} \tag{2.8}$$

The number l_ψ is the conformal weight of the field $\psi(x)$.

In order to construct conformal invariant theories one introduces a Lorentz scalar field $\sigma(x)$ with the anomalous transformation law (Isham *et al.*, 1970)

$$\sigma'(x') = \sigma(x) - \frac{1}{4f}\ln\left|\det\left(\frac{\partial x'}{\partial x}\right)\right| \tag{2.9}$$

where f is a constant.

We assume that the fields of the nine pseudoscalar mesons, and the eight scalar mesons which belong to the $SU(3)$ octet, transform under conformal transformations as the $\psi(x)$ of equations (2.7) with

$$l_P = l_S = 0 \tag{2.10}$$

while the $SU(3)$ singlet scalar field transforms as the $\sigma(x)$ of equation (2.9). This means that under conformal transformations we get

$$M \to M' = \left|\det\left(\frac{\partial x'}{\partial x}\right)\right|^{-1/4} M \tag{2.11}$$

The above equation implies that the action of the Lagrangian L_{KE} of equation (2.4) is invariant under conformal transformations.

The weights of equation (2.10) differ from the canonical weights $l_P = l_S = -1$. This leaves no problems since in a theory which contains the field $\sigma(x)$ the conformal weight of a field has no invariant significance. The Lagrangian L_{KE} gives to lowest order in f the kinetic energy part of the free Lagrangian of the nonet of scalar and the nonet of pseudoscalar mesons

$$L_{KE} = -\tfrac{1}{2}\mathrm{Tr}\,(\partial_\mu S\,\partial_\mu S + \partial_\mu P\,\partial_\mu P) - f\,\mathrm{Tr}\,[S(\partial_\mu S\,\partial_\mu S + \partial_\mu P\,\partial_\mu P)] + \frac{f^2}{24}\mathrm{Tr}\,(4\partial_\mu P\,\partial_\mu P^3 - 3\partial_\mu P^2\,\partial_\mu P^2) + \ldots \tag{2.12}$$

The higher order terms in the above expansion represent derivative type interaction terms. To complete the free Lagrangian mass terms must be added. Consider the Lagrangians

$$L_1 = \frac{1}{2f^2}\mathrm{Tr}\,[(MM^+MM^+ + M^+MM^+M)(\alpha_1 + \beta_1\lambda_8)] = \frac{1}{f^2}\mathrm{Tr}\,[[1 + 4fS + 8f^2S^2 + \tfrac{32}{3}f^3S^3 + \ldots](\alpha_1 + \beta_1\lambda_8) + \tfrac{2}{3}f^3(2PSP - [S,P^2]_+)\beta_1\lambda_8] + O(f^2) \tag{2.13}$$

$$L_2 = -\frac{1}{2f^2}\mathrm{Tr}\,(MM^+)\,\mathrm{Tr}\,[(MM^+ + M^+M)(\alpha_2 + \beta_2\lambda_8)] = \frac{1}{f^2}[9\alpha_2 + 6f\,\mathrm{Tr}\,[S\,.(2\alpha_2 + \beta_2\lambda_8)] + 6f^2\,\mathrm{Tr}\,[S^2(2\alpha_2 + \beta\lambda_8)] + 4\alpha_2 f^2\,\mathrm{Tr}\,S\,\mathrm{Tr}\,S + 4\beta_2 f^2\,\mathrm{Tr}\,S.\mathrm{Tr}\,(S\lambda_8) + 8\alpha_2 f^3\,\mathrm{Tr}\,S^3 + \beta_2 f^3\,\mathrm{Tr}\,[4S^3 + 2PSP - [S,P^2]_+]\lambda_8 + 8\alpha_2 f^3\,\mathrm{Tr}\,S\,\mathrm{Tr}\,S^2 + 4\beta_2 f^3\,[\mathrm{Tr}\,S\,\mathrm{Tr}\,(S^2\lambda_8) + \mathrm{Tr}\,(S\lambda_8)\,\mathrm{Tr}\,S^2]] + O(f^2) \tag{2.14}$$

$$L_3 = \frac{1}{2f^2}\mathrm{Tr}\,[(MM^+ + M^+M)(\alpha_3 + \beta_3\lambda_8)] = \frac{1}{f^2}\mathrm{Tr}\,[(1 + 2fS + 2f^2S^2 + \tfrac{4}{3}f^3S^3 + \ldots)(\alpha_3 + \beta_3\lambda_8) + \frac{f^3}{3}(2PSP - [S,P^2]_+)\beta_3\lambda_8] + O(f^2) \tag{2.15}$$

$$L_4 = \frac{1}{2f^2}\mathrm{Tr}\,[(M + M^+)(\alpha_4 + \beta_4\lambda_8)] = \frac{1}{f^2}\mathrm{Tr}\,[[1 + fS + \frac{f^2}{2}(S^2 - P^2) + \frac{f^3}{6}(S^3 - PSP - [S,P^2]_+) + \frac{f^4}{24}P^4 + \ldots](\alpha_4 + \beta_4\lambda_8)] \tag{2.16}$$

$$L_5 = \frac{\alpha_5}{4f^2}\mathrm{Tr}\,(M - M^+)\,\mathrm{Tr}\,(M - M^+) + \frac{\beta_5}{4f^2}\mathrm{Tr}\,(M - M^+)\,\mathrm{Tr}\,[(M - M^+)\lambda_8] = -\alpha_5[\mathrm{Tr}\,P\,\mathrm{Tr}\,P + 2f\,\mathrm{Tr}\,P\,\mathrm{Tr}\,(SP) - \frac{f^2}{2}\mathrm{Tr}\,P\,\mathrm{Tr}\,P^3 + \ldots] - \beta_5[\mathrm{Tr}\,P\,.\mathrm{Tr}\,P\lambda_8 + f\,\mathrm{Tr}\,(P\lambda_8)\,\mathrm{Tr}\,(SP) + \frac{f}{2}\mathrm{Tr}\,P\,\mathrm{Tr}([S,P]_+\lambda_8) - \frac{f^2}{6}\,\mathrm{Tr}\,(P\lambda_8)\,.\mathrm{Tr}\,P^3 - \frac{f^2}{6}\mathrm{Tr}\,P\,\mathrm{Tr}\,P^3\lambda_8 + \ldots] \tag{2.17}$$

We take

$$L_{\mathrm{total}} = L_{KE} + \sum_{i=1}^{5} L_i \tag{2.18}$$

The Lagrangians L_1 and L_2 have conformal weights –4. The conformal weights of the other Lagrangians L_3, L_4 and L_5 are –2, –1, and –2 respectively. For $\beta_1 = \beta_2 = \beta_3 = 0$ the Lagrangians L_1, L_2 and L_3 are invariant under the group $SU(3)_L \otimes SU(3)_R$. The terms proportional to β_1, β_2 and β_3 are invariant only under $SU(2)_L \otimes SU(2)_R$ transformations. The braking of the $SU(3)_L \otimes SU(3)_R$ symmetry has been introduced through the matrix λ_8. The L_4 breaks the chiral symmetry. The reason for the introduction of this term is that we want the model to satisfy generalised PCAC. The L_5 also breaks chiral symmetry.

To order f^0 our Lagrangian is required to be the free Lagrangian of a nonet of scalar mesons and a nonet of pseudoscalar mesons all of which have their physical masses. The coefficients α_i and β_i, $i = 1,\ldots,5$, are determined in such a way that the mesons of the model get their physical masses and in addition the coefficients of the terms $\operatorname{Tr} S$ and $\operatorname{Tr} S\lambda_8$ vanish. The constant term appearing in equation (2.18) is eliminated by the addition of a c-number term. The $I = 0$ member of the $SU(3)$ octet and the $SU(3)$ singlet which is a member of the nonet are mixed. We define the mixing angles by the relations

$$\begin{aligned} S_0 &= \varepsilon \sin \Theta_s + \varepsilon' \cos \Theta_s \\ S_8 &= \varepsilon \cos \Theta_s - \varepsilon' \sin \Theta_s \end{aligned} \tag{2.19}$$

$$\begin{aligned} P_0 &= \eta \sin \Theta_P + \eta' \cos \Theta_P \\ P_8 &= \eta \cos \Theta_P - \eta' \sin \Theta_P \end{aligned} \tag{2.20}$$

The η' is identified with the $\eta'(958)$.

Let δ, K', $\bar{K}'$, ε and ε' be the members of the nonet of scalar mesons, where δ has $I = 1$, K' and $\bar{K}'$ have $I = \frac{1}{2}$ and ε and ε' have $I = 0$. Then the above requirements imply

$$\begin{aligned}
\alpha_1 &= \tfrac{1}{24}[(m_\pi{}^2 + 2m_K{}^2) - (m_\delta{}^2 + 2m_{K'}^2)] \\
\alpha_2 &= \tfrac{1}{72}(m_\delta{}^2 + 2m_{K'}^2) - \tfrac{1}{24}(m_{\varepsilon'}^2 \cos^2 \Theta_s + m_\varepsilon{}^2 \sin^2 \Theta_s) \\
\alpha_3 &= \tfrac{1}{4}[m_{\varepsilon'}^2 \cos^2 \Theta_s + m_\varepsilon{}^2 \sin^2 \Theta_s - (m_\pi{}^2 + 2m_K{}^2)] \\
\alpha_4 &= \tfrac{1}{3}(m_\pi{}^2 + 2m_K{}^2) \\
\alpha_5 &= \tfrac{1}{18}[3(m_{\eta'}^2 \cos^2 \Theta_p + m_\eta{}^2 \sin^2 \Theta_p) - (m_\pi{}^2 + 2m_K{}^2)] \\
\beta_1 &= \frac{1}{4\sqrt{3}}[(m_\pi{}^2 - m_K{}^2) - (m_\delta{}^2 - m_{K'}^2)] \\
\beta_2 &= \frac{1}{24\sqrt{6}}[3(m_{\varepsilon'}^2 - m_\varepsilon{}^2)\sin 2\Theta_s + 4\sqrt{(2)}\,(m_\delta{}^2 - m_{K'}^2)] \\
\beta_3 &= \frac{1}{8\sqrt{6}}[3(m_{\varepsilon'}^2 - m_\varepsilon{}^2)\sin 2\Theta_s + 12\sqrt{(2)}\,(m_\pi{}^2 - m_K{}^2)] \\
\beta_4 &= \frac{2}{\sqrt{3}}(m_\pi{}^2 - m_K{}^2) \\
\beta_5 &= -\frac{1}{6\sqrt{6}}[3(m_{\eta'}^2 - m_\eta{}^2)\sin 2\Theta_P + 4\sqrt{(2)}\,(m_\pi{}^2 - m_K{}^2)]
\end{aligned} \tag{2.21}$$

The absence of terms of the form $\mathrm{Tr}(S\lambda_8)\mathrm{Tr}(S\lambda_8)$ and $\mathrm{Tr}(P\lambda_8)\mathrm{Tr}(P\lambda_8)$ imply that the mixing angles are given by the relations

$$\sin^2\Theta_s = \frac{\frac{1}{3}(4m_{K'}^2 - m_\delta^2) - m_\varepsilon^2}{m_{\varepsilon'}^2 - m_\varepsilon^2}, \tag{2.22}$$

$$\sin^2\Theta_P = \frac{\frac{1}{3}(4m_K^2 - m_\pi^2) - m_\eta^2}{m_{\eta'}^2 - m_\eta^2} \tag{2.23}$$

i.e. they are not free parameters in this formalism.

Considering the transformation

$$M \to e^{i\varepsilon_\alpha\lambda_\alpha} M e^{i\varepsilon_\alpha\lambda_\alpha}$$

and applying the method of Gell-Mann and Lévy (1960) we can calculate the axial vector current $J_{5\mu}^\alpha$ and its divergence. We find

$$\partial_\mu J_{5\mu}^\alpha = \frac{\sqrt{2}}{f} m_\alpha^2 P^\alpha + (\text{higher order terms}) \tag{2.24}$$

where

$$\begin{aligned} m_\alpha &= m_\pi \qquad \text{for } \alpha = 1,2,3 \\ &= m_K \qquad \text{for } \alpha = 4,5,6,7 \end{aligned} \tag{2.25}$$

Therefore the model satisfies generalized PCAC.

We are interested in calculating the decay widths of the scalar mesons into two pseudoscalar mesons. The Lagrangian $L(SPP)$ contributes to such processes. From equations (2.18) and (2.21) we get:

$$\begin{aligned} L(SPP) = & -f\,\mathrm{Tr}\,(S\,\partial_\mu P\,\partial_\mu P) - f\,\mathrm{Tr}\,(PSP\,[\tfrac{1}{18}(m_\pi^2 + 2m_K^2) \\ & + \frac{1}{\sqrt{3}}[(m_\pi^2 - m_K^2) + \tfrac{2}{3}(m_\delta^2 - m_{K'}^2)]\,\lambda_8]) - f\,\mathrm{Tr}\,([S, P^2]_+ \\ & \times [\tfrac{1}{18}(m_\pi^2 + 2m_K^2) - \frac{1}{3\sqrt{3}}(m_\delta^2 - m_{K'}^2)\,\lambda_8]) \\ & - \frac{\sqrt{(3)}f}{9}[3(m_{\eta'}^2\cos^2\Theta_P + m_\eta^2\sin^2\Theta_P) \\ & - (m_\pi^2 + 2m_K^2)]\,P_0\,\mathrm{Tr}\,(SP) + \frac{f}{6\sqrt{6}}[3(m_{\eta'}^2 - m_\eta^2)\sin 2\Theta_P \\ & + 4\sqrt{(2)}\,(m_\pi^2 - m_K^2)][\sqrt{(2)}\,P_8\,\mathrm{Tr}(SP) + \frac{\sqrt{3}}{2}P_0\,\mathrm{Tr}\,([S,P]_+\lambda_8)] \end{aligned} \tag{2.26}$$

Explicitly we find for the derivatives containing Lagrangian

$$\begin{aligned} \frac{1}{f}L(SPP)_D = & -\frac{1}{\sqrt{2}}\boldsymbol{\delta}\cdot\partial_\mu\bar{K}\boldsymbol{\tau}\,\partial_\mu K - \frac{1}{\sqrt{2}}\partial_\mu\boldsymbol{\pi}\cdot(\partial_\mu\bar{K}\boldsymbol{\tau}K' + \bar{K}'\,\boldsymbol{\tau}\,\partial_\mu K) \\ & - \frac{1}{\sqrt{3}}\left(\sin\Theta_s + \frac{\cos\Theta_s}{\sqrt{2}}\right)\varepsilon\,\partial_\mu\boldsymbol{\pi}\,.\,\partial_\mu\boldsymbol{\pi} - \frac{1}{\sqrt{3}}\left(\cos\Theta_s - \frac{\sin\Theta_s}{\sqrt{2}}\right) \\ & \times \varepsilon'\,\partial_\mu\boldsymbol{\pi}\,.\,\partial_\mu\boldsymbol{\pi} - \frac{2}{\sqrt{3}}\left(\sin\Theta_P + \frac{\cos\Theta_P}{\sqrt{2}}\right)\boldsymbol{\delta}\cdot\partial_\mu\boldsymbol{\pi}\,\partial_\mu\eta \end{aligned}$$

$$
\begin{aligned}
&-\frac{2}{\sqrt{3}}\left(\cos\Theta_P-\frac{\sin\Theta_P}{\sqrt{2}}\right)\boldsymbol{\delta}.\partial_\mu\boldsymbol{\pi}\,\partial_\mu\eta' \\
&-\frac{1}{\sqrt{3}}\left(2\sin\Theta_s-\frac{\cos\Theta_s}{\sqrt{2}}\right)\varepsilon\,\partial_\mu\bar{K}\partial_\mu K \\
&-\frac{1}{\sqrt{3}}\left(2\cos\Theta_s+\frac{\sin\Theta_s}{\sqrt{2}}\right)\varepsilon'\,\partial_\mu\bar{K}\partial_\mu K \\
&-\frac{1}{\sqrt{3}}\left(2\sin\Theta_P-\frac{\cos\Theta_P}{\sqrt{2}}\right)(\bar{K}'\,\partial_\mu K+\partial_\mu\bar{K}K')\,\partial_\mu\eta \\
&-\frac{1}{\sqrt{3}}\left(2\cos\Theta_P+\frac{\sin\Theta_P}{\sqrt{2}}\right)(\bar{K}'\,\partial_\mu K+\partial_\mu\bar{K}K')\,\partial_\mu\eta' \\
&-\frac{1}{\sqrt{3}}S_0\,\partial_\mu P_0\,\partial_\mu P_0-\frac{2}{\sqrt{3}}S_8\,\partial_\mu P_0\,\partial_\mu P_8 \\
&-\frac{1}{\sqrt{3}}\left(S_0-\frac{S_8}{\sqrt{2}}\right)\partial_\mu P_8\,\partial_\mu P_8 \qquad (2.27)
\end{aligned}
$$

where $K=\begin{pmatrix}K^+\\K^0\end{pmatrix}$ and for simplicity the last three terms of the above Lagrangian have not been expressed in terms of the physical fields ε, ε', η and η'.

The non-derivative part of the Lagrangian $L(SPP)$ is given by

$$
\begin{aligned}
\frac{1}{f}L(SPP)_{N-D}=&\frac{1}{2\sqrt{2}}[m_\pi^2-2m_K^2+\tfrac{4}{3}(m_\delta^2-m_{K'}^2)]\,\boldsymbol{\delta}.\bar{K}\boldsymbol{\tau}K \\
&-\frac{1}{6\sqrt{2}}[3m_\pi^2+2(m_\delta^2-m_{K'}^2)]\boldsymbol{\pi}.(\bar{K}'\,\boldsymbol{\tau}K+\bar{K}\boldsymbol{\tau}K') \\
&-\frac{m_\pi^2}{2\sqrt{3}}\left(\sin\Theta_s+\frac{\cos\Theta_s}{\sqrt{2}}\right)\varepsilon\,\boldsymbol{\pi}.\boldsymbol{\pi}-\frac{m_\pi^2}{2\sqrt{3}}\left(\cos\Theta_s-\frac{\sin\Theta_s}{\sqrt{2}}\right) \\
&\times\varepsilon'\,\boldsymbol{\pi}.\boldsymbol{\pi}\;-\frac{1}{6\sqrt{6}}\left[6\sqrt{(2)}\,m_\pi^2\left(\sin\Theta_P+\frac{\cos\Theta_P}{\sqrt{2}}\right)\right. \\
&+[6\sqrt{(2)}\,(m_{\eta'}^2\cos^2\Theta_P+m_\eta^2\sin^2\Theta_P)-3(m_{\eta'}^2-m_\eta^2) \\
&\times\sin 2\Theta_P+2\sqrt{(2)}\,(m_\pi^2-m_K^2)]\sin\Theta_P \\
&-\sqrt{(2)}\,[3(m_{\eta'}^2-m_\eta^2)\sin 2\Theta_P+4\sqrt{(2)}\,(m_\pi^2-m_K^2)] \\
&\left..\cos\Theta_P\right]\boldsymbol{\delta}.\boldsymbol{\pi}\eta-\frac{1}{6\sqrt{6}}\left[6\sqrt{(2)}\,m_\pi^2\left(\cos\Theta_P-\frac{\sin\Theta_P}{\sqrt{2}}\right)\right. \\
&+[6\sqrt{(2)}\,(m_{\eta'}^2\cos\Theta_P{}^2+m_\eta^2\sin^2\Theta_P) \\
&-3(m_{\eta'}^2-m_\eta^2)\sin\Theta_P+2\sqrt{(2)}\,(m_\pi^2-m_K^2)]\cos\Theta_P \\
&+\sqrt{(2)}\,[3(m_{\eta'}^2-m_\eta^2)\sin 2\Theta_P \\
&\left.+4\sqrt{(2)}\,(m_\pi^2-m_K^2)]\sin\Theta_P\right]\boldsymbol{\delta}.\boldsymbol{\pi}\eta' \\
&-\frac{1}{2\sqrt{6}}[2\sqrt{(2)}\,m_K^2\sin\Theta_s-[3m_\pi^2-2m_K^2 \\
&+4(m_\delta^2-m_{K'}^2)]\cos\Theta_s]\,\bar{K}K\varepsilon-\frac{1}{2\sqrt{6}}[2\sqrt{(2)}\,m_K^2\cos\Theta \\
&+[3m_\pi^2-2m_K^2+4(m_\delta^2-m_{K'}^2)]\sin\Theta_s]\,\bar{K}K\varepsilon'
\end{aligned}
$$

$$
\begin{aligned}
&-\frac{1}{12\sqrt{6}}[3[4\sqrt{(2)}(m_{\eta'}^2\cos^2\Theta_P+m_\eta{}^2\sin^2\Theta_P)\\
&+(m_{\eta'}^2-m_\eta{}^2)\sin 2\Theta_P]\sin\Theta_P+2[m_\pi{}^2-4m_K{}^2\\
&+6(m_\delta{}^2-m_{K'}^2)-3\sqrt{(2)}(m_{\eta'}^2-m_\eta{}^2)\sin\Theta_P]\cos\Theta_P]\\
&\times(\bar{K}'K+\bar{K}K')\eta-\frac{1}{12\sqrt{6}}[3[4\sqrt{(2)}(m_{\eta'}^2\cos^2\Theta_P\\
&+m_\eta{}^2\sin^2\Theta_P)+(m_{\eta'}^2-m_\eta{}^2)\sin 2\Theta_P]\ \cos\Theta_P-2[m_\pi{}^2\\
&-4m_K{}^2+6(m_\delta{}^2-m_{K'}^2)-3\sqrt{(2)}(m_{\eta'}^2-m_\eta{}^2)\sin 2\Theta_P]\\
&\times\sin\Theta_P](\bar{K}'K+KK')\eta'-\frac{1}{6\sqrt{3}}[[6(m_{\eta'}^2\cos^2\Theta_P\\
&+m_\eta{}^2\sin^2\Theta_P)-(m_\pi{}^2+2m_K{}^2)]S_0-[3(m_{\eta'}^2-m_\eta{}^2)\\
&\times\sin 2\Theta_P+2\sqrt{(2)}(m_\pi{}^2-m_K{}^2)]S_8]P_0P_0\\
&+\frac{1}{6\sqrt{6}}[\sqrt{(2)}(m_\pi{}^2-4m_K{}^2)S_0+[3\sqrt{(2)}(m_{\eta'}^2-m_\eta{}^2)]\\
&\times\sin 2\Theta_P+3m_\pi{}^2]S_8]P_8P_8+\frac{1}{6\sqrt{6}}[[6\sqrt{(2)}(m_{\eta'}^2-m_\eta{}^2)\\
&\times\sin 2\Theta_P+8(m_\pi{}^2-m_K{}^2)]S_0\\
&-[6\sqrt{(2)}(m_{\eta'}^2\cos^2\Theta_P+m_\eta{}^2\sin^2\Theta_P)\\
&+3(m_{\eta'}^2-m_\eta{}^2)\sin 2\Theta_P]S_8]P_0P_8
\end{aligned}
\tag{2.28}
$$

The Lagrangian $L(SPP)$ of equation (2.26) is the sum of the Lagrangians $L(SPP)_D$ and $L(SPP)_{N-D}$ of equations (2.27) and (2.28).

3. *Decay Widths of Scalar Mesons and Meson–Meson Scattering Lengths*

From the Lagrangians (2.27) and (2.28) the decay widths of the scalar mesons into two pseudoscalar mesons can be calculated. We shall be interested in the decays $\varepsilon\to\pi\pi$, $\varepsilon'\to\pi\pi$, $\delta\to\pi\eta$, $\varepsilon'\to\bar{K}K$, and $K'\to K\pi$. We find

$$
\Gamma(\varepsilon\to\pi\pi)=f^2\left(\sin\Theta_s+\frac{\cos\Theta_s}{\sqrt{2}}\right)^2\frac{(m_\varepsilon{}^2-m_\pi{}^2)^2(m_\varepsilon{}^2-4m_\pi{}^2)^{1/2}}{32\pi m_\varepsilon{}^2}
\tag{3.1}
$$

$$
\Gamma(\varepsilon'\to\pi\pi)=f^2\left(\cos\Theta_s-\frac{\sin\Theta_s}{\sqrt{2}}\right)^2\frac{(m_{\varepsilon'}^2-m_\pi{}^2)^2(m_{\varepsilon'}^2-4m_\pi{}^2)^{1/2}}{32\pi m_{\varepsilon'}^2}
\tag{3.2}
$$

$$
\begin{aligned}
\Gamma(\delta\to\pi\eta)=\frac{f^2}{48\pi m_\delta{}^3}&\left[(m_\delta{}^2-m_\eta{}^2)\left(\sin\Theta_P+\frac{\cos\Theta_P}{\sqrt{2}}\right)+[m_{\eta'}^2\cos^2\Theta_P\right.\\
&+m_\eta{}^2\sin^2\Theta_P-m_\pi{}^2-\frac{1}{\sqrt{2}}\sin\Theta_P\cos\Theta_P(m_{\eta'}^2-m_\eta{}^2)]\\
&\times\sin\Theta_P-[\sin\Theta_P\cos\Theta_P(m_{\eta'}^2-m_\eta{}^2)\\
&\left.+\frac{2\sqrt{2}}{3}(m_\pi{}^2-m_K{}^2)]\cos\Theta_P\right]^2([m_\delta{}^2-(m_\eta+m_\pi)^2]\\
&\times[m_\delta{}^2-(m_\eta-m_\pi)^2])^{1/2}
\end{aligned}
\tag{3.3}
$$

$$\Gamma(\varepsilon' \to \bar{K}K) = \frac{f^2}{192\pi m_{\varepsilon'}^2}[2\sqrt{(2)}(m_{\varepsilon'}^2 - m_K{}^2)\cos\Theta_s + [m_{\varepsilon'}^2 - 4m_K{}^2 + 3m_\pi{}^2 + 4(m_\delta{}^2 - m_{K'}^2)]\sin\Theta_s]^2(m_{\varepsilon'}^2 - 4m_K{}^2)^{1/2} \quad (3.4)$$

$$\Gamma(K' \to K\pi) = \frac{f^2(m_{K'}^2 + 2m_\delta{}^2 - 3m_K{}^2)^2}{384\pi m_{K'}^3} \times ([m_{K'}^2 - (m_K + m_\pi)^2][m_{K'}^2 - (m_K - m_\pi)^2])^{1/2} \quad (3.5)$$

The parameter f enters in the axial vector current. If we omit the S fields the kinetic energy term (2.4), from which the axial vector current is calculated, is identical to the kinetic energy term of Cronin (1967). The value of f found in this paper from the leptonic decays of pseudoscalar mesons is

$$f = 2m_\pi^{-1} \quad (3.6)$$

We shall assume that f has this value.

Several papers (Particle Data Group, 1971) have indicated the existence of an isosinglet scalar meson with mass around 700 MeV, which decays into two pions, and its width is much larger than 100 MeV. The following values of the mass and the width are suggested (Ebel *et al.*, 1971)

$$m_\varepsilon = 700 \pm 200 \text{ MeV}, \qquad \Gamma_\varepsilon = 500 \pm 300 \text{ MeV} \quad (3.7)$$

We shall assume that our ε meson has mass 700 MeV. Our second isosinglet scalar meson ε' will be identified (Particle Data Group, 1971) with the $\eta_0{}^+(1070)$ (or S^*). The total width of this resonance which decays to $\pi\pi$ (partial fraction less than 65) and to $\bar{K}K$ (partial fraction more than 35) is

$$\Gamma_{\varepsilon'} = 150\text{–}300 \text{ MeV} \quad (3.8)$$

The isotriplet scalar meson δ will be identified (Particle Data Group, 1971) with the $\pi_N(975)$, whose decay width is 58 ± 11 MeV. The $I = \frac{1}{2}$ scalar mesons are controversial. Analysis of the K_{13} form factors indicates a $K\pi$ resonance around 1 GeV (Pati & Sebastian, 1968).

To calculate the widths we must know the mixing angle Θ_s. If the mass $m_{K'}$ is known the angle Θ_s can be calculated from equation (2.22). Also if the widths $\Gamma(\varepsilon \to \pi\pi)$ or $\Gamma(\varepsilon' \to \pi\pi)$ are known experimentally the angle Θ_s can be calculated from equations (3.1) or (3.2) respectively. But this is not the case. To give an estimate of the widths predicted by the model we assume that $\Gamma(\varepsilon' \to \pi\pi) = 140$ MeV. Then a solution of equation (3.2) is

$$\Theta_s = 66^\circ\, 45', \quad (3.9)$$

which will be used in the subsequent calculations.

From equations (2.22) and (3.9) we get

$$m_{K'} \simeq 1000 \text{ MeV} \quad (3.10)$$

A solution of equation (2.23) which will be used in the calculation is

$$\Theta_P = -11^\circ \quad (3.11)$$

From equations (3.1), (3.3)–(3.6), and (3.9)–(3.11) we find:

$$\Gamma(\varepsilon \to \pi\pi) = 840 \text{ MeV} \tag{3.12}$$

$$\Gamma(\varepsilon' \to \bar{K}K) = 135 \text{ MeV} \tag{3.13}$$

$$\Gamma(\delta \to \pi\eta) = 180 \text{ MeV} \tag{3.14}$$

$$\Gamma(K' \to K\pi) = 580 \text{ MeV} \tag{3.15}$$

The values of $\Gamma(\varepsilon \to \pi\pi)$ and $\Gamma(\varepsilon' \to \bar{K}K)$ are more or less consistent with the experimental data, while the width $\Gamma(\delta \to \pi\eta)$ is larger than the experimental width. Probably we should not identify our $I=1$ scalar meson with the meson $\pi_N(975)$.

The meson–meson scattering lengths can be easily calculated. The direct interaction terms of our Lagrangian give the scattering lengths of Cronin (1967). In addition there is a contribution from scalar meson exchange. The S-wave π–π scattering lengths are given by the expressions

$$\alpha_0(\pi\pi) = \frac{7}{64\pi} f^2 m_\pi + \frac{f^2 m_\pi^{\,3}}{16\pi}\left(\frac{g^2_{\varepsilon\pi\pi}}{m_\varepsilon^{\,2}} + \frac{g^2_{\varepsilon'\pi\pi}}{m^2_{\varepsilon'}}\right) + \frac{27 f^2 m_\pi^{\,3}}{32\pi}\left(\frac{g^2_{\varepsilon\pi\pi}}{m_\varepsilon^{\,2} - 4m_\pi^{\,2}} + \frac{g^2_{\varepsilon'\pi\pi}}{m^2_{\varepsilon'} - 4m_\pi^{\,2}}\right) \tag{3.16}$$

$$\alpha_1(\pi\pi) = 0 \tag{3.17}$$

$$\alpha_2(\pi\pi) = -\frac{1}{32\pi} f^2 m_\pi + \frac{f^2 m_\pi^{\,3}}{16\pi}\left(\frac{g^2_{\varepsilon\pi\pi}}{m_\varepsilon^{\,2}} + \frac{g^2_{\varepsilon'\pi\pi}}{m^2_{\varepsilon'}}\right) \tag{3.18}$$

We find $\alpha_0(\pi\pi) \simeq 0{\cdot}16 m_\pi^{-1}$, and $\alpha_2(\pi\pi) \simeq -0{\cdot}04 m_\pi^{-1}$. The contribution of the ε and ε' exchange graphs to $\pi\pi$ scattering lengths is very small.

The S-wave πK scattering lengths can be similarly calculated. We find

$$\alpha_{1/2}(\pi K) = \frac{1}{8\pi(m_\pi + m_K)}\left\{ f^2 m_\pi m_K + \frac{m_\pi^{\,2} g_{\varepsilon\pi\pi}(g_{\varepsilon KK} m_K^{\,2} - g'_{\varepsilon KK})}{2m_\varepsilon^{\,2}} + \frac{m_\pi^{\,2} g_{\varepsilon'\pi\pi}(g_{\varepsilon' KK} m_K^{\,2} - g'_{\varepsilon' KK})}{2m^2_{\varepsilon'}} - \frac{f^2[3(m_\pi - m_K)^2 + 2(m^2_{K'} - m_\delta^{\,2}) + 3m_K^{\,2}]^2}{72[(m_\pi - m_K)^2 + m^2_{K'}]} + \frac{f^2[3(m_\pi + m_K)^2 + 2(m^2_{K'} - m_\delta^{\,2}) + 3m_K^{\,2}]^2}{24[(m_\pi + m_K)^2 + m^2_{K'}]}\right\} \tag{3.19}$$

$$\alpha_{3/2}(\pi K) = \frac{1}{8\pi(m_\pi + m_K)}\left\{ -\frac{f^2 m_\pi m_K}{2} + \frac{m_\pi^{\,2} g_{\varepsilon\pi\pi}(g_{\varepsilon KK} m_K^{\,2} - g'_{\varepsilon KK})}{2m_\varepsilon^{\,2}} + \frac{m_\pi^{\,2} g_{\varepsilon'\pi\pi}(g_{\varepsilon' KK} m_K^{\,2} - g'_{\varepsilon' KK})}{2m^2_{\varepsilon'}} + \frac{f^2[3(m_\pi - m_K)^2 + 2(m^2_{K'} - m_\delta^{\,2}) + 3m_K^{\,2}]^2}{36[(m_\pi - m_K)^2 + m^2_{K'}]}\right\} \tag{3.20}$$

The above expressions give $\alpha_{1/2}(\pi K) \simeq 0{\cdot}19 m_\pi^{-1}$ and $\alpha_{3/2}(\pi K) \simeq -0{\cdot}05 m_\pi^{-1}$. The main contribution comes from the contact term.

Also the S-wave KK scattering lengths can be calculated. We find

$$\alpha_0(KK) = 0 \tag{3.21}$$

$$\alpha_1(KK) = \frac{1}{16\pi m_K}\left\{-\frac{f^2 m_K{}^2}{2} + \frac{(g_{\delta KK} m_K{}^2 - g'_{\delta KK})^2}{m_\delta{}^2} + \frac{(g_{\varepsilon KK} m_K{}^2 - g'_{\varepsilon KK})^2}{m_\varepsilon{}^2} + \frac{(g_{\varepsilon' KK} m_K{}^2 - g'_{\varepsilon' KK})^2}{m_{\varepsilon'}^2}\right\} \tag{3.22}$$

from which we get $\alpha_1(KK) \simeq -0{\cdot}13 m_\pi^{-1}$. Again the contribution coming from scalar meson exchange is small. The couplings $g_{\varepsilon\pi\pi}$, $g_{\varepsilon'\pi\pi}$, $g_{\varepsilon KK}$, $g_{\varepsilon' KK}$ and $g_{\delta KK}$ appearing in some of Eqs. (3.16)–(3.22) are obtained from the Lagrangian (2.27), and the couplings $g'_{\varepsilon KK}$, $g'_{\varepsilon' KK}$ and $g'_{\delta KK}$ from the Lagrangian (2.28).

4. *Meson–Baryon Lagrangians*

We assign the baryons to the representation $(3^*, 3)$ and $(3, 3^*)$ of $SU(3)_L \otimes SU(3)_R$ as was originally suggested by Gell-Mann (1964) because it gives a D-type axial-vector current, while the representation $(8,1)$ and $(1,8)$ gives pure F-type currents. The F-type admixture of the axial-vector current seems to be only of the order of 30% (Gabbibo, 1963). We use the two component Weyl field (Marshak *et al.*, 1965). Therefore, if $B_\alpha{}^\beta$ is the baryon field we write

$$B_\alpha{}^\beta = \begin{pmatrix} (B_R)_\alpha^{\beta'} \\ (B_L)_{\alpha'}^{\beta} \end{pmatrix} \tag{4.1}$$

where

$$B_L = \tfrac{1}{2}(1+\gamma_5) B \sim (3_L{}^*, 3_R) \tag{4.2}$$

$$B_R = \tfrac{1}{2}(1-\gamma_5) B \sim (3_L, 3_R{}^*) \tag{4.3}$$

We consider the baryon kinetic energy term

$$\begin{aligned} L'_{\text{KE}} &= -\tfrac{1}{2}(B_L{}^+)_\alpha{}^\beta \gamma_4 \gamma_\mu \overleftrightarrow{\partial}_\mu (B_L)_\beta{}^\alpha - \tfrac{1}{2}(B_R{}^+)_\alpha{}^\beta \gamma_4 \gamma_\mu \overleftrightarrow{\partial}_\mu (B_R)_\beta{}^\alpha \\ &= -\tfrac{1}{2}\operatorname{Tr}(\bar{B}\gamma_\mu \overleftrightarrow{\partial}_\mu B) \end{aligned} \tag{4.4}$$

and the following meson–baryon Lagrangians:

$$\begin{aligned} L^1_{\text{MB}} &= \varepsilon_{\alpha\gamma\delta}\, \varepsilon^{\beta\zeta\eta}(B_L{}^+)_\zeta{}^\gamma \gamma_4 M_\beta{}^\alpha (B_R)_\eta{}^\delta + \varepsilon_{\alpha\gamma\delta}\, \varepsilon^{\beta\zeta\eta}(B_R{}^+)_\zeta{}^\gamma \gamma_4 (M^+)_\beta{}^\alpha (B_L)_\eta{}^\delta \\ &= -\operatorname{Tr}(\bar{B}B) + 3\bar{B}_0 B_0 + f[3\sqrt{(3)}\,\bar{B}_0 B_0 S_0 - \sqrt{(3)}\operatorname{Tr}(\bar{B}B) S_0 \\ &\quad - \sqrt{(3)}\operatorname{Tr}(\bar{B}S) B_0 + \operatorname{Tr}(\bar{B}\{S, B\}_+) - \sqrt{(3)}\,\bar{B}_0 \operatorname{Tr}(SB)] \\ &\quad + if[-3\sqrt{(3)}\,\bar{B}_0 \gamma_5 B_0 P_0 + \sqrt{(3)}\operatorname{Tr}(\bar{B}\gamma_5 B) P_0 + \sqrt{(3)}\operatorname{Tr}(\bar{B}\gamma_5 P) B_0 \\ &\quad - \operatorname{Tr}(\bar{B}\gamma_5\{P, B\}_+) + \sqrt{(3)}\,\bar{B}_0 \gamma_5 \operatorname{Tr}(PB)] + \frac{f^2}{2}[\operatorname{Tr}(\bar{B}B)\operatorname{Tr}(P^2) \\ &\quad - \operatorname{Tr}(\bar{B}\{P^2, B\}_+)] + O(f^2) \end{aligned} \tag{4.5}$$

$$
\begin{aligned}
L^2_{\mathrm{MB}} &= \varepsilon_{\alpha\gamma\delta}\,\varepsilon^{\beta\zeta\eta}(B_L{}^+)_\zeta{}^\gamma\gamma_4 M_\beta{}^\alpha(B_R\lambda_8)^\delta_\eta + \varepsilon_{\alpha\gamma\delta}\,\varepsilon^{\beta\zeta\eta}(B_R{}^+)_\zeta{}^\gamma\gamma_4(M^+)_\beta{}^\alpha(B_L\lambda_8)_\eta{}^\delta \\
&\quad + \varepsilon_{\alpha\gamma\varepsilon}\,\varepsilon^{\beta\zeta\eta}(\lambda_8 B_L{}^+)_\zeta{}^\gamma\gamma_4 M_\beta{}^\alpha(B_R)_\eta{}^\delta + \varepsilon_{\alpha\gamma\delta}\,\varepsilon^{\beta\zeta\eta}(\lambda_8 B_R{}^+)_\zeta{}^\gamma\gamma_4(M^+)_\beta{}^\alpha(B_L)_\eta{}^\delta \\
&= -\mathrm{Tr}\,(\bar{B}B\lambda_8) + \surd(6)\,\bar{B}_0 B_8 + f\,[3\surd(2)\bar{B}_0 B_8 S_0 - \surd(3)\,\mathrm{Tr}\,(\bar{B}B\lambda_8)\,S_0 \\
&\quad - \surd(2)\,\mathrm{Tr}\,(\bar{B}S)\,B_8 - \surd(3)\,\mathrm{Tr}\,(\bar{B}S\lambda_8)\,B_0 + \mathrm{Tr}\,(\bar{B}\{S, B\lambda_8\}_+)] \\
&\quad + if\,[-3\surd(2)\,\bar{B}_0\gamma_5 B_8 P_0 + \surd(3)\,\mathrm{Tr}\,(\bar{B}\gamma_5 B\lambda_8)\,P_0 \\
&\quad + \surd(2)\,\mathrm{Tr}\,(\bar{B}\gamma_5 P)\,B_8 + \surd(3)\,\mathrm{Tr}\,(\lambda_8\,\bar{B}\gamma_5 P)\,B_0 \\
&\quad - \mathrm{Tr}\,(\bar{B}\gamma_5\{P, B\lambda_8\}_+)] + \frac{f^2}{2}\,[\mathrm{Tr}\,(\bar{B}B\lambda_8)\,\mathrm{Tr}\,P^2 - \mathrm{Tr}\,(\bar{B}\{P^2, B\lambda_8\}_+)] \\
&\quad + Hc + O(f^2)
\end{aligned}
\tag{4.6}
$$

$$
\begin{aligned}
L^3_{\mathrm{MB}} &= \varepsilon_{\alpha\gamma\delta}\,\varepsilon^{\beta\zeta\eta}(B_L{}^+)_\zeta{}^\gamma\gamma_4 M_\beta{}^\alpha(\lambda_8 B_R)_\eta{}^\delta + \varepsilon_{\alpha\gamma\delta}\,\varepsilon^{\beta\zeta\eta}(B_R{}^+)_\zeta{}^\gamma\gamma_4(M^+)_\beta{}^\alpha(\lambda_8 B_L)_\eta{}^\delta \\
&\quad + \varepsilon_{\alpha\gamma\delta}\,\varepsilon^{\beta\zeta\eta}(B_L{}^+\lambda_8)_\zeta{}^\gamma\gamma_4 M_\beta{}^\alpha(B_R)_\eta{}^\delta + \varepsilon_{\alpha\gamma\delta}\,\varepsilon^{\beta\zeta\eta}(B_R{}^+\lambda_8)_\zeta{}^\gamma\gamma_4(M^+)_\beta{}^\alpha(B_L)_\eta{}^\delta \\
&= -\mathrm{Tr}\,(\bar{B}\lambda_8 B) + \surd(6)\,\bar{B}_0 B_8 + f\,[3\surd(2)\,\bar{B}_0 B_8 S_0 - \surd(3)\,\mathrm{Tr}\,(\bar{B}\lambda_8 B)\,S_0 \\
&\quad - \surd(2)\,\mathrm{Tr}\,(\bar{B}S)\,B_8 - \surd(3)\,\mathrm{Tr}\,(\bar{B}\lambda_8 S)\,B_0 + \mathrm{Tr}\,(\bar{B}\{S, \lambda_8 B\}_+)] \\
&\quad + if\,[-3\surd(2)\,\bar{B}_0\gamma_5 B_8 P_0 + \surd(3)\,\mathrm{Tr}\,(\bar{B}\gamma_5\lambda_8 B)\,P_0 \\
&\quad + \surd(2)\,\mathrm{Tr}\,(\bar{B}\gamma_5 P)\,B_8 + \surd(3)\,\mathrm{Tr}\,(\bar{B}\lambda_8\gamma_5 P)\,B_8 - \mathrm{Tr}\,(\bar{B}\gamma_5\{P, \lambda_8 B\}_+)] \\
&\quad + \frac{f^2}{2}\,[\mathrm{Tr}\,(\bar{B}\lambda_8 B)\,\mathrm{Tr}\,P^2 - \mathrm{Tr}\,(\bar{B}\{P^2, \lambda_8 B\}_+)] + Hc + O(f^2)
\end{aligned}
\tag{4.7}
$$

$$
\begin{aligned}
L^4_{\mathrm{MB}} &= \mathrm{Tr}\,(B_L{}^+ M^+)\,\gamma_4\,\mathrm{Tr}\,(M^+ B_R) + \mathrm{Tr}\,(B_R{}^+ M)\,\gamma_4\,\mathrm{Tr}\,(MB_L) \\
&= 3\bar{B}_0 B_0 + \surd(3)\,f\,[\bar{B}_0\,\mathrm{Tr}\,(SB) + \mathrm{Tr}\,(\bar{B}S)\,B_0] + i\surd(3)\,f\,[\bar{B}_0\gamma_5\,\mathrm{Tr}\,(SB) \\
&\quad + \mathrm{Tr}\,(\bar{B}S)\,\gamma_5 B_0] + O(f^2)
\end{aligned}
\tag{4.8}
$$

In the expansion of the Lagrangians L^i_{MB}, $i = 1,\ldots,4$ we have kept only those terms of order $O(f^2)$ which contribute to πN scattering. The Lagrangian L^1_{MB} is invariant under the group $SU(3)_L \otimes SU(3)_R$. The L^1_{MB} alone implies that the $SU(3)$ singlet of the baryon nonet has negative mass, which is in absolute value twice as large as the mass of the other baryons. A negative mass baryon is interpreted as a baryon with positive mass and opposite parity (Freund & Nambu, 1964). This will be identified with the $Y_0{}^*(1405)$. The Lagrangians L^2_{MB} and L^3_{MB} break the $SU(3)_L \otimes SU(3)_R$ symmetry and allow the octet of baryons to get their physical masses, as we shall see in a moment. Since the mass of the $Y_0{}^*(1405)$ is quite a bit lower than twice the average mass of the octet, the Lagrangian L^4_{MB} is introduced. If we assign the baryons the canonical weight $l_B = -\frac{3}{2}$ the Lagrangian densities L'_{KE}, L^1_{MB}, L^2_{MB} and L^3_{MB} transform as scalar densities of conformal weight $l = -4$, while the L^4_{MB} transforms as scalar density of weight $l = -5$.

Consider the Lagrangian

$$
L_{\mathrm{MB}} = L'_{\mathrm{KE}} + \sum_{i=1}^{4} C_i L^i_{\mathrm{MB}}
\tag{4.9}
$$

The coefficients C_i of the above expression are determined by the requirement that the L_{MB} to zero order in f, will be the free Lagrangian of a nonet of baryons all of which have their physical masses apart from electromagnetic mass splitting (which we ignore). The higher order in f terms of L_{MB} are pseudoscalar meson–baryon and scalar meson–baryon interaction terms. From (4.9) we get to zero order in f:

$$-\tfrac{1}{2}\operatorname{Tr}(\bar{B}\gamma_\mu \overleftrightarrow{\partial_\mu} B) - C_1\left(\sum_{i=1}^{8}\bar{B}_i B_i - 2\bar{B}_0 B_0\right) - C_2[2\operatorname{Tr}(\bar{B}B\lambda_8) - \sqrt{(6)}(\bar{B}_0 B_8 + \bar{B}_8 B_0) - C_3[2\operatorname{Tr}(\bar{B}\lambda_8 B) - \sqrt{(6)}(\bar{B}_0 B_8 + \bar{B}_8 B_0)] + 3C_4\bar{B}_0 B_0 \quad (4.10)$$

The above expression has off-diagonal terms of the type $\bar{B}_0 B_8 + \bar{B}_8 B_0$ and must be diagonalized. Therefore, we assume $B_0 - B_8$ mixing with a mixing angle Θ_B according to

$$\begin{aligned} \Lambda &= \cos\Theta_B B_8 + \sin\Theta_B B_0 \\ Y' &= -\sin\Theta_B B_8 + \cos\Theta_B B_0 \end{aligned} \quad (4.11)$$

As we argued before the physical $\frac{1}{2}^-$ isosinglet $Y_0^*(1405)$ will be represented by the field $\gamma_5 Y'$. From (4.11) and the assumption that the expression (4.10) is the free Lagrangian of a nonet of baryons we find

$$\begin{aligned} C_1 &= \frac{m_\Sigma + m_N + m_\Xi}{3} \\ C_2 &= \frac{m_\Sigma - m_N}{2\sqrt{3}} \\ C_3 &= \frac{m_\Sigma - m_\Xi}{2\sqrt{3}} \\ C_4 &= \frac{m_Y - m_\Lambda - m_\Sigma}{3} \end{aligned} \quad (4.12)$$

As expected the parameters C_2 and C_3 are of the order of Δm. The mixing angle Θ_B is given by

$$\tan 2\Theta_B = -\sqrt{(2)}\,\frac{2m_\Sigma - m_N - m_\Xi}{3m_Y + 4(m_N + m_\Xi) - 2m_\Sigma - 3m_\Lambda} = -0{\cdot}0226 \quad (4.13)$$

We find $\Theta_B = 39'$, a very small mixing angle.

From the Lagrangian L_{MB} of equations (4.9) we get, if we use equations (4.12),

$$L_{\varepsilon NN} = -\frac{m_N f}{\sqrt{3}}\left(\sin\Theta_s + \frac{\cos\Theta_s}{\sqrt{2}}\right)\varepsilon\bar{N}N \quad (4.14)$$

$$L_{\varepsilon' NN} = -\frac{m_N f}{\sqrt{3}}\left(\cos\Theta_s - \frac{\sin\Theta_s}{\sqrt{2}}\right)\varepsilon'\bar{N}N \quad (4.15)$$

which give the coupling constants $G_{\varepsilon NN}$ and $G_{\varepsilon' NN}$.

We define the coupling constants $G_{\varepsilon\pi\pi}$ and $G_{\varepsilon'\pi\pi}$ by the formulas

$$\Gamma(\varepsilon \to \pi\pi) = \frac{3G^2_{\varepsilon\pi\pi}}{16\pi}\left(\frac{m_\varepsilon^2 - 4m_\pi^2}{4}\right)^{1/2} \tag{4.16}$$

$$\Gamma(\varepsilon' \to \pi\pi) = \frac{3G^2_{\varepsilon'\pi\pi}}{16\pi}\left(\frac{m^2_{\varepsilon'} - 4m_\pi^2}{4}\right)^{1/2} \tag{4.17}$$

Comparing the above expressions with the widths $\Gamma(\varepsilon \to \pi\pi)$ and $\Gamma(\varepsilon' \to \pi\pi)$ we get

$$G^2_{\varepsilon\pi\pi} = \frac{f^2}{3m_\varepsilon^2}\left(\sin\Theta_s + \frac{\cos\Theta_s}{\sqrt{2}}\right)^2 (m_\varepsilon^2 - m_\pi^2)^2 \tag{4.18}$$

$$G^2_{\varepsilon\pi\pi} = \frac{f^2}{3m^2_{\varepsilon'}}\left(\cos\Theta_s - \frac{\sin\Theta_s}{\sqrt{2}}\right)^2 (m^2_{\varepsilon'} - m_\pi^2)^2 \tag{4.19}$$

From equations (4.14), (4.15), (4.18) and (4.19) we find the ratios

$$R = \frac{|G_{\varepsilon NN}|}{|G_{\varepsilon\pi\pi}|} = \frac{m_N m_\varepsilon}{m_\varepsilon^2 - m_\pi^2} = 1{\cdot}4 \tag{4.20}$$

$$R' = \frac{|G_{\varepsilon' NN}|}{|G_{\varepsilon'\pi\pi}|} = \frac{m_N m_{\varepsilon'}}{m^2_{\varepsilon'} - m_\pi^2} = 0{\cdot}89 \tag{4.21}$$

The above ratios are independent of the parameters f and Θ_s. The values of R and R' found from the coupling of ε and ε' to the trace of the energy-momentum tensor are (Genz & Steiner, 1971a, b)

$$R = \frac{m_\varepsilon m_N}{m_\varepsilon^2 + 2m_\pi^2} = 1{\cdot}24 \tag{4.22}$$

$$R' = \frac{m_{\varepsilon'} m_N}{m^2_{\varepsilon'} + 2m_\pi^2} = 0{\cdot}85 \tag{4.23}$$

The above values of R and R' are very close to the corresponding values of these ratios of equations (4.20) and (4.21).

From equations (4.14) and (4.18) the value of the product $|G_{\varepsilon\pi\pi} G_{\varepsilon NN}|/4\pi$ can be calculated. We find

$$\frac{|G_{\varepsilon\pi\pi} G_{\varepsilon NN}|}{4\pi} = \left(\sin\Theta_s + \frac{\cos\Theta_s}{\sqrt{2}}\right)^2 \frac{f^2(m_\varepsilon^2 - m_\pi^2)\, m_N}{12\pi m_\varepsilon} = 4{\cdot}9 \tag{4.24}$$

The value of the above product found by Engels (1970) is $5{\cdot}49 \pm 0{\cdot}32$. Petersen and Pišut (1972) give the values $5{\cdot}68 \pm 1{\cdot}0$ if the $I = J = 0$ $\pi\pi$ phase shift $\delta_0{}^0$ is down–up or up–up, and $4{\cdot}06 \pm 0{\cdot}8$ if $\delta_0{}^0$ is down–down or up–down. Also Schaile (Ebel *et al.*, 1971) gives the value 5·0 and Strauss (Ebel *et al.*, 1971) the value 4·0. Thus expression (4.24) is in reasonable agreement with experiment.

Also we find

$$\frac{|G_{\varepsilon'\pi\pi} G_{\varepsilon'NN}|}{4\pi} = \left(\cos\Theta_s - \frac{\sin\Theta_s}{\sqrt{2}}\right)^2 \frac{f^2(m_{\varepsilon'}^2 - m_\pi{}^2)\, m_N}{12\pi m_{\varepsilon'}} = 0{\cdot}34 \quad (4.25)$$

This product is very sensitive to the value of the angle Θ_s. From the coupling of ε' to the trace of the energy-momentum tensor we get (Genz & Steiner, 1971b)

$$\frac{|G_{\varepsilon'\pi\pi} G_{\varepsilon'NN}|}{4\pi} = \frac{\Gamma_{\text{tot}}}{6{\cdot}2m} \quad (4.26)$$

where Γ_{tot} is the total width of ε'. In our model we have $\Gamma_{\text{tot}} = 275$ MeV and equation (4.26) gives

$$\frac{|G_{\varepsilon'\pi\pi} G_{\varepsilon'NN}|}{4\pi} = 0{\cdot}32 \quad (4.27)$$

in reasonable agreement with expression (4.25). The experimental estimates of the above product are contradictory (Genz & Steiner, 1971b).

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